FACTORS INFLUENCING THE ROUNDING OF SAND GRAINS

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INTRODUCTION

In 1910, while discussing the rounding of sand grains with Professor A. W. Grabau, it seemed to the author that the influence of viscosity was not sufficiently emphasized in the literature on that subject. Subsequent discussion with Professor C. P. Berkey suggested this investigation. The thanks of the author are due to Professor James F. Kemp, and especially to Professor C. P. Berkey for many kind and valuable suggestions.

THE MOLECULAR FORCES OF LIQUIDS

For a clear understanding of the forces acting on a particle submerged in water it is necessary that we review briefly a few of the elementary definitions of physics. This can most clearly be done by means of an illustration.

If we look carefully at the surface of a glass of water, we notice that it is not horizontal but curves upward at the sides of the containing vessel as though attracted by it. If we dip a clean glass rod in water and remove it, we shall see adhering to it a thin film of water. Upon slightly shaking the rod this film will be dis-

lodged and in falling will assume a more or less spherical form. The smaller the drop, the more perfect its spherical shape. Here we have a homely demonstration of the forces acting on the liquid. The creep-up of the water on the sides of the glass is due to the attraction of the glass for the water; the drop of water remaining on the glass rod is held there by the same force, that is—adhesion. In falling, the water from the rod does not fly off in a series of small particles, but assumes a spherical shape because the component particles of water, or, in other words, its molecules are attracted toward each other. This is cohesion. Adhesion is the attraction of unlike molecules for each other; cohesion is the attraction exhibited between molecules of the same substance.[1] The force due to the cohesion of the molecules of different substances and that due to the adhesion between the molecules of different substances varies. The cohesion of water is less than its adhesion for glass, hence the glass rod is enabled to tear away a certain amount of water.[2] If, however, we dip a glass rod into mercury and withdraw it, nothing will adhere, because, in this case, cohesion is the stronger force.

The space through which cohesion is active is the "sphere of molecular attraction." It is a sphere about 0.00005 mm. in diameter.[3] If we now assume that a liquid is made up of a number of layers of molecules, we will see that the top layer, the free surface, will be attracted unequally because part of its "sphere of molecular attraction" lies outside the liquid.[4]

In Fig. 1 xy is the surface of the liquid. A and B represent two molecules in the surface and beneath the surface respectively.

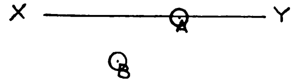

FIG. 1

The circles surrounding them represent the "sphere of molecular attraction." The molecule B is attracted equally in all directions by the molecules falling within its sphere; in the case of the molecule A, however, the attraction will be downward, as the attracting molecules only occupy that part of the sphere lying within

[1] Nichols and Franklin, *Elements of Physics*, 124.

[2] F. Pockels in Winkelman's *Handbuch der Physik*, I, 882.

[3] Duff, *Textbook of Physics*, 146. [4] *Ibid.*, 147.

the water. On this account the surface of the liquid is in a state of tension, and in order to move the molecule B to the surface we would have to overcome this force. We may liken the condition of the surface of the liquid to that of the stretched rubber membrane of a ball. We have a pressure at right angles to the surface, capillary pressure, causing a tension parallel to the surface, surface tension.[1]

Let us now consider a grain submerged in a liquid and let us note the action of the different forces upon it. The body will be pulled down by the force of gravity, the magnitude of the pull being determined by the difference in the specific gravity of the solid and the liquid. If we consider water, then the force will be equal to $vg\ (d-1)$; where v is the volume, g the acceleration due to gravity, and d the density of the solid.

In moving through the liquid, the grain will carry down a thin film of water held by adhesion. There is a certain friction developed in this movement which will not be friction between the grain and the water, but friction of water with water. The friction developed by a thin film of water sliding on water is "superficial viscosity." The term "skin friction" is also applied to it.[2] This is the friction especially considered in the flow of water through pipes and conduits. In addition, through the downward movement of the grain, the shape of the liquid is disturbed. Any disturbance or change of shape in a liquid calls forth a resistance, "viscosity." But even if the particle were moving in a "perfect fluid," i.e., a fluid without any viscosity, its energy would gradually be dissipated in forming waves.[3]

To summarize then, a body moving through water must overcome resistance due to three causes; (1) viscosity, (2) skin-friction, and (3) wave-resistance.

If we take a case in which the liquid has a definite velocity, the conditions as outlined above will not change. In this case the grain will be acted on by a force which is the resultant of the velocity and gravity, and will have the direction of the diagonal

[1] Ganot, *Physics*, 122.

[2] Basset, *Elementary Treatise on Hydrodynamics*, 52.

[3] *Ibid.*, 51.

of the parallelogram of forces constructed with velocity and gravity as sides (Fig. 2). The grain will experience no resistance in the direction of the velocity, as it will simply move along with the water. The downward movement will experience the same resistance as though the liquid were at rest.

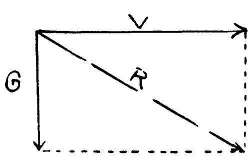

FIG. 2

MOLECULAR FORCES AND TRANSPORTATION

Sediment is transported by water in one of three methods. It is either floated on the surface, or rolled along the bottom, or carried in suspension.

Small grains, when carefully sifted over the surface of water, float, due to the fact that their weight is not sufficient to overcome the surface tension of water. Since surface tension may be defined as the "force tending to make a liquid contract to the smallest area admissible," it will have the tendency to drive the floating grains together.[1] This apparent attraction of grains into patches, although not explained, has been noted by James C. Graham and F. W. Simonds, who described this method of sand-transportation as occurring on the Connecticut and Llanos rivers respectively.[2] Experiments carried on by Simonds seem to show that if the launching be favorable, about 40 per cent of the component grains of most sandstones will float on water. Floating patches of sand and dust have been noticed by the author on the Iowa and Cedar rivers on still days during the summer, where they look essentially like floating patches of scum or foam, and also on the quiet water along the shore of the North Sea, near Otterndorf and Cuxhaven in Germany. While the condition necessary for the transportation by flotation are somewhat unusual, this method still appears to have more importance than is usually attributed to it.

The floating of the grain depends on two molecular forces, viz., cohesion and adhesion. Cohesion causes the tension in the free surface of the water, and resists all attempts to break this surface. Adhesion serves as a modifying factor. If the adhesion

[1] Duff, *op. cit.*, 146.

[2] Graham, *A.J.S.*, series 3, XL, 476; Simonds, *Am. Geol.*, XVII, 29.

between the grain and water be strong enough to wet the grain, it will sink at once; if adhesion be weak, the grain will remain dry and float. The adhesion between the grain and the water may be entirely destroyed by coating them with oil. The so-called "oil-flotation process" of ore dressing depends to a great extent on this principle. The finely pulverized ore is mixed with a small quantity of oil. The metallic sulphides, such as galena, chalcopyrite, and sphalerite, have strong adhesion for oil, and are readily coated, while the quartz and other gangue remain free, unless an excessive amount of oil is used. When the ore is allowed to slide into the settling tanks, the gangue sinks readily, but the coated sulphides float off. Here it seems that molecular forces cause flotation rather than the decrease in specific gravity due to the combined weight of oil and mineral. As the specific gravity of the oil taken is approximately 0.8, in the case of galena the volume of oil to mineral would have to be in the ratio of 32 to 1, to bring the density of the combined material down to that of water.[1]

Sharp, angular grains float more readily than those of spherical shape. This is due to the fact that the force due to the surface tension increases with an increase in the surface area exposed to it. The more nearly spherical a grain, the smaller the ratio between the surface area and the mass of the grain, and hence the greater the ratio of weight to surface tension. Irregularity of shape increases the ratio of surface to mass, and hence decreases the tendency to break through the surface of the film.

The power of water to carry material in suspension depends on a number of factors, some of which are: the shape, size, and composition of the particles; the viscosity, composition, and velocity of the water; the presence of colloids; the character of the river bottom; the course of the stream, etc. The size of grain carried depends directly on the velocity. The more irregular the shape, the greater will be the resistance encountered in settling. The presence of colloidal substances causes rapid settling.[2] Again there may be a change in the composition of the water causing an interaction with the sediment, such as the precipitation of alumina

[1] Adams, *M. and Sc. Press*, May 7, 1904, etc.

[2] F. W. Clarke, *Data of Geochemistry*, 430 (Bull. 330, U.S.G.S.).

by the carbonates of calcium and magnesium, and a consequent settling of the silt.[1] The presence of salts, alkalies, and acids in solution hasten the rate of precipitation. However, Wheeler arrives at the conclusion that there is practically no difference in the rate of settlement of sand and silt in salt and fresh water.[2] When the particles are very fine, as mud and ooze, the rate of settlement is slightly faster in salt than in fresh water. Others have shown that settling is far more rapid in salt than in fresh water, and attribute this fact to a chemical interaction between the salt water and the sediment, carried in this case as a colloid.[3] There is reason to doubt this explanation, and the more rapid settling in salt water seems to be due to a decrease in the viscosity of the water.[4] Rough and irregular river bottoms and swinging meanders tend to keep the water in a stirred condition and hence aid in holding material.

METHODS OF ROUNDING

Sand grains are reduced in size by collision and friction. Hence we know that the wear of a grain depends on a number of factors, such as hardness, weight, distance of travel, cleavage, tenacity, velocity of movement, etc. The rounding of sand grains under the varying conditions has been ably discussed from the geological standpoint by McKee[5] and Goodchild.[6] The movements of solids through fluids have been investigated from the mathematical standpoint especially by Basset[7] and Allen.[8] This feature has also been noted to some extent by Blake,[9] Walther,[10] and Barrell.[11]

[1] E. W. Hilgaard, *A.J.S.*, 1873, p. 288; 1879, p. 205.

[2] W. H. Wheeler, *Nature*, June 20, 1901.

[3] See F. W. Clarke, *Bull. 330, U.S.G.S.*, and H. S. Allen, *Nature*, July 18, 1901, for bibliographies.

[4] J. F. Blake, *Geol. Mag.*, Decade IV, Vol. X, 12; W. B. Scott, *Introduction to Geology*, 141; Carl Barus, *Bull. 36, U.S.G.S.*: Chamberlin and Salisbury, *College Geology*, 316.

[5] McKee, *Edinburgh Geol. Soc.*, VII, 296.

[6] Goodchild, *ibid.*, 208.

[7] Basset, *Elementary Treatise on Hydrodynamics.*

[8] Allen, *Phil. Mag.*, 1900.

[9] Blake, *Geol. Mag.*, Decade IV, Vol. X, 12.

[10] Walther, *Das Gesetz der Wustenbildung.*

[11] Barrell, *Jour. Geol.* (1908), XVI, 159.

Summary of previous work.—McKee in his work evolves the formula

$$R \propto \frac{\text{size} \times \text{specific gravity} \times \text{distance traveled}}{\text{hardness}}$$

where R is the rounding (or the wear).

Considering a cube with the edge x, the distance traveled would be roughly proportionate to the number of times the grain turned over, hence $\frac{D}{4x}$ could be placed instead of distance. The weight of the cube would be x^3 Sp. Gr.

Substituting in the above equation we have

$$R \propto \frac{x^3 \text{ Sp. Gr.} \frac{d}{4x}}{\text{hardness}}$$

reducing to

$$R \propto \frac{x^2 \text{ Sp. Gr. } d}{4h}$$

Or in more general terms—

$$R \propto \frac{x^2 \text{ Sp. Gr. } d}{mh}$$

where m is a constant depending on the shape of the grain. m is 4 in the case of a cube, 3.1416 in the case of a sphere, etc. If the grain is under water allowance must be made and

$$R \propto \frac{x^2 \cdot (\text{Sp. Gr.} - 1) \cdot d}{mh}$$

Goodchild goes farther and determines a general limiting condition to the wear taking place. His work may be summarized as follows:

Since the sand is completely surrounded by a film of the water in which it is submerged, it will be acted on by surface tension. By decreasing the size of a particle we increase the ratio of area to volume, and hence to weight. Since the surface tension of water will act over the area exposed, its magnitude compared to the weight of the grain will increase with decrease in size. Finally, he assumes that a balance between weight and surface tension will be reached, such that no further rupture of the film of water surrounding the grain can take place, and hence all wear will

cease. Thus Goodchild concludes that the factor limiting the amount of wear possible on submerged bodies, is surface tension.

Experimental work.—As stated before, in the movement of bodies through water resistance due to three causes must be

EXPERIMENTS

mm. Diam.	Glycerin	Water	Alcohol
Cassiterite (6.4)*			
3–2	Collision	Collision	Collision
2–1½	Collision	Collision	Collision
1¼–¾	? Repulsion ?	Collision	Collision
ca. ½	Repulsion	? Collision ?	Collision
<½	Repulsion	Repulsion	? Repulsion ?
Chromite (4.5)			
3–2	Repulsion	Collision	Collision
2–1½	Repulsion	? Collision ?	Collision
1–¾	Strong Repulsion	Repulsion	Collision
ca. ½	Strong Repulsion	Repulsion	? Repulsion ?
<½	Strong Repulsion	Repulsion	? Repulsion ?
Quartz (2.65)			
3–2	Collision	Collision	Collision
2–1½	? Repulsion ?	Collision	Collision
1–½	Repulsion	Repulsion	? Repulsion ?
<½	Repulsion	Repulsion	? Repulsion ?
Gypsum (2.35)			
3–2	Collision	Collision	Collision
2–1½	? Repulsion ?	Collision	Collision
1¼–¾	? Repulsion ?	Repulsion	? Collision ?
ca. ½	? Repulsion ?	Repulsion	? Repulsion ?
<½	? Repulsion ?	Repulsion	Repulsion
Anthracite (1.6)			
3–2¼	....	Collision	Collision
2–1½	....	? Collision ?	Collision
1½–1	....	Repulsion	Repulsion
¾–½	....	Repulsion	Repulsion
<½	....	Repulsion	Repulsion

* The figures beside the minerals represent specific gravity.

DATA

	Sp. Gravity	Surf. Tension	Viscosity
Glycerin	1.252	66.5	8.0
Water	1.0	71.	.10
Alcohol	.887	23.4	.011

overcome, viz., viscosity, skin-friction, and wave-resistance. Unless these three factors are overcome, grains cannot collide.

The effect of surface tension, however, is one aiding wear, since it tends to draw grains together in its effort to force the water to assume the least area permissible under the conditions to which it is subject. Thus viscosity, since it is the most potent of the three factors mentioned, limits the minimum size to which wear takes place. The energy of the particle must overcome the viscosity to allow collision. Since the velocity of different grains in water is roughly equivalent, their energy varies directly with the size, the larger grains only having enough power to overcome viscosity. In the case of small grains the water acts as a cushion preventing actual collision, or checking the velocity of contact. To show the action of viscosity in preventing collisions of grains the following experiments were performed.

Grains were dropped down long glass tubes filled with liquids of different viscosities, and the action at the meeting of the grains was observed. Grains of different specific gravities were taken so as to overcome the difference in the specific gravities of the liquids.

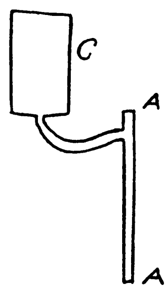

Fig. 3

Again an experiment was performed (Fig. 3) in which the glycerin was allowed to run from the reservoir *C* through the tube *AA* down which the different grains were dropped. The results were practically identical with those above.

It will be noted that the surface tensions of water and glycerin are nearly the same, but that the viscosities are in the ratio of eighty to one. In the case of glycerin it was apparently impossible for the grains of small diameter to collide. Whenever a larger grain would overtake a smaller and slower falling one, there was an apparent repulsion between the two as they were held apart by the viscosity. In small and light grains the repulsion appeared violent so that often a clearing space of a quarter of an inch was shown by grains that apparently were going to collide. As can be seen from the table, in the case of water the protection against collision was much less. Small grains of quartz, less than 1 mm. in diameter showed fairly strong repulsion, but above that size collisions were the rule. Again in the case of alcohol, with a surface tension of

23.4 and a viscosity of 0.011, repulsion was only noticed in the finest grains.

SUMMARY

The results of these experiments seem to show that viscosity is the factor protecting grains from wear. Viscosity will not only prevent the wear of the smaller grains, but it will also act as a buffer and will greatly lessen the velocity of grains when about to collide with each other or with the bottom of the river. In view of the results it seems improbable to the writer that grains less than 0.75 mm. in diameter could be well rounded under water. Well-rounded grains of about this and smaller diameter appear to be the result of wind work, in which case the protecting factor, viscosity, would be practically zero, so that there would be no limit to the minimum size attainable by wear.